PSEUDO-PUBLIC GOODS
AND URBAN DEVELOPMENT:
A Game Theoretic Model
of Local Public Goods

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ABSTRACT: In his 1956 article, “A Pure Theory of Local Expenditures,” Charles Tiebout introduced the foundation of what was to become the competitive market model of analyzing local government service provision and choice of residential location. This model has since served as the base of a large body of work describing and evaluating the impact of the actions and structure of local government. Recent philosophical and ideological trends in government have led to many services and amenities traditionally provided directly by local governments being supplied instead by various privatized arrangements. This article presents a game theoretic model of local public good provision that takes into account the impact of privatized service delivery. The model develops a framework for analyzing the public/private decision process and the integration effects of these decisions.

In his 1956 article, “A Pure Theory of Local Expenditures,” Charles Tiebout introduced the foundation of what was to become the competitive market model of analyzing local government service provision, taxation decisions, and choice of residential and business locations. This model has since been the base of a massive body of work describing the ideal role, actions, and structure of government emerging from the assumptions inherent in Tiebout’s article. This article begins with the fundamental assumptions of the Tiebout model and its later elaborations. The framework will provide the basis for a new perspective for analyzing the provision of local public goods. Specifically, this article seeks to present a formal game theoretical model through which the Tiebout logic can address

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new developments in the privatized provision of local amenities and the scope of inquiry can be brought more closely in tune with the modern reality of development and government in urban areas.

The Tiebout Model

Tiebout’s initial market model of local expenditures and service provision is based on a number of simple assumptions about the nature of local-level government services, the local environment, and the consumer-voters who live in the region. First, the model assumes that local public goods (LPGs) are true public goods in the traditional economic definition: That is, they are both nonrival and nonexcludable. The nonrival quality implies that their enjoyment or consumption by any one individual in their preferred manner has no negative impact on the ability of any other individual to enjoy the same good. Nonexcludability means that once the good is provided to any one member of the society (here the locality within a metropolitan area or system), no other member can reasonably be prevented from consuming the good (Rosen, 1992).

Recognizing the inherent difficulty in directly determining the preferred level of consumption (and thus spending) on each of these types of goods by each individual, Tiebout hypothesizes that individual (or household) preferences over packages of LPGs are revealed by locational decisions when a residential (or by extension, business) move is made. Governments will likewise use the information revealed by the cumulative set of these locational decisions across a society to determine appropriate changes to their package of LPGs and revenue-generating mechanisms.

Assumptions of the Tiebout Model

In his model, Tiebout hypothesizes the seven minimal conditions that are necessary for the desired efficiency results to come about. They are: (1) Consumer-voters are fully mobile and will move to that community where their preference patterns, which are set, are best satisfied. (2) Consumer-voters are assumed to have full knowledge of differences among revenue and expenditure patterns and to react to these differences. (3) There are a large number of communities in which the consumer-voters may choose to live. (4) Restrictions due to employment opportunities are not considered, and it may be assumed that all persons are living on dividend income. (5) The public services supplied exhibit no external economies or diseconomies between communities. (6) For every pattern of community services set by a city following the preferences of its older residents, there is an optimal community size. (7) Communities seek to adjust their population size in order to attain and maintain the optimal size for their package of services (Tiebout, 1956).

Assumptions 1 and 2 imply that consumer-voters are rational, utility-maximizing actors with full and complete information. Assumption 3 implies a competitive marketplace in which no city has monopoly or oligopoly power over consumers. As a result, each city-firm is a “price-taker” competing simply through the quality of its LPG package. Assumption 4 removes the constraints of employment opportunities and their locations from the housing decisions of consumer-voters. As such, this assumption grants individuals complete mobility between communities, as required by assumption 1. Assumption 5 simply assumes that no externalities, positive or negative, are present in the market. The
final two assumptions deal with the issue of economies of scale. Assumption 6 implies that for each distinct LPG package, there is some population level and related level of production at which the entire package can be produced for the lowest average cost per resident. This also assumes the provision process is already as efficient as possible in its internal functioning. It is important to note that this does not require all or even any of the individual goods provided by the local government to be produced at their most efficient quantity, merely at that level that produces the lowest overall cost for the total package (see Ostrom, 1972). When combined with the attempts to reach this population size, which is embodied in assumption 7, the local community approximates the profit-maximizing firm of the general economic model. It attempts to raise the number of consumer-voters it serves, at a given tax level, to the point where its costs are minimized and then seeks to prevent any more customers from purchasing its services.

Implications of the Tiebout Model

A number of implications have been drawn about the local level provision of services from the hypothetical framework hypothesized by Tiebout. Most important for the purposes of this discussion are those dealing with spatial urban structure.

From the Tiebout model, a clear role emerges for market mechanisms in designing urban environments. If the theory is accepted, it is clear that the stronger the competitive forces acting through the locational decisions of residents in a metropolitan area, the more efficient local government will be. The greater this efficiency becomes, the more satisfied individuals will be with the performance of their local governments. Two dominant lines of thought have emerged as to the appropriate form of governmental behavior necessary to meet this theoretical mandate.

Decentralization Approach

The first model is the decentralization approach. In such an arrangement, the responsibility for providing LPGs throughout a metropolitan area is broken down into a wide number of geographical units, each of which has the exclusive right to serve the population within its bounds. This pattern follows most directly from the Tiebout formulation and is very representative of the suburbanization trend begun in the United States in the postwar era (Ostrom, Tiebout, & Warren, 1961; Tiebout, 1956). Its efficiency benefits rest firmly on the model of competition described by the assumptions of the Tiebout hypothesis. Specific weaknesses of this approach will be dealt with later.

“Clubs” and “Fiscal Federalism”

The second general model is usually referred to as either the “club” or “fiscal federalism” approach (Oates, 1972; Olson, 1969). The term “fiscal federalism,” in this case, refers to divisions of fiscal and policy authority within the local level of government (Hochman, Pines, & Thisse, 1995) rather than the more common meaning associated with the work of Paul Peterson (1981, 1995). The line of reasoning behind this model asserts that a specific level or organization of government should be created to deal with each service that is to be provided. The sizes of these clubs would be limited to the most efficient number for production of the particular good or service (Ostrom, 1972). Many special governmental authorities, such as a mass transit district, are approximations of
this approach. From a market perspective, the strength of this method is that it allows
individuals to “contract” with a club or organization for the ideal level of service they
desire or to abstain entirely if the service is not of interest. This reduces the possibility of
suboptimal individual outcomes resulting from aggregate group decisions. Moreover, the
problem of LPGs being produced in quantities that are not the most efficient for the in-
dividual good is avoided.

On the negative side, this solution does little or nothing to reduce the underprovision of
goods with positive externalities (such as education) inherent in all pure individual-level
market scenarios. Since those benefits are not received by the purchaser of the LPG, he or
she has little incentive to pay for them without some sort of centralized subsidy—a tra-

Another fundamental difficulty with such a model is that it is nonspatial—no fixed loca-
tions for either producer or consumer are considered, and no costs of transporting either
the good to the consumer or the consumer to the production/consumption location are
included. When the costs of transportation (e.g., busing vs. walking to school) and/or the
potential degradation in service quality over greater distances (e.g., the fire protection
from a fire station) are added to this model, the economy of scale benefits of a “club”
approach could well be offset by geographic considerations in many cases (Hochman,
Pines, & Thisse, 1995; Thisse & Wildasin, 1992). In general, this approach seems not to
have received as much serious consideration as the decentralization model, perhaps in
recognition of those difficulties detailed above.

EXPANDING THE TIEBOUT MODEL

Over the years, critics have found fault with the rigidity or unrealistic nature of many
of the central assumptions of the Tiebout model. Tiebout himself acknowledges the un-
realistic nature of some of the assumptions throughout his work (Tiebout, 1956, pp. 420,
423). Many potential key elements of residential locational decisions are also omitted
from the basic Tiebout formulation. The combination of these flaws prevents this tradi-
tion of research from harnessing its full analytical potential and leaves it blind to a num-
ber of the central issues of modern urban politics. By relaxing a subset of these restrictive
assumptions in a game theoretic framework and including selected factors not discussed
by Tiebout, this article seeks to expand the applicability of his logic to a broad number of
modern issues and situations.

Relaxing the Tiebout Assumptions

The first of the major flaws inherent in the specification of the Tiebout model is the
requirement of costless perfect mobility on the part of individuals. This premise is clearly
suspect on two grounds. First, moving from residence to residence usually involves sub-
stantial direct and indirect costs.

Indirect Costs

If all governments, their LPG packages, and the tax systems used to finance the LPGs
were equal, there would be little information that needed to be collected and analyzed
about a new residential location. However, such a situation would destroy the entire premise of the Tiebout model (as well as conflict with reality). In any metropolitan area of even modest size, the mere act of determining which available package is best becomes extremely difficult as the number of municipalities and policy variations increases. Information costs are linked directly to this difficulty. Even after these information hurdles are cleared, the consumer-voter must choose among the variety of residential units within the chosen community, further increasing costs (Lowery & Lyons, 1989; Lyons & Lowery, 1986; Teske, Schneider, Mintrom, & Best, 1993).

**Direct Costs**

After overcoming the indirect expenses to moving, direct costs remain. These include the fees paid for legal and financial services and the costs of physically transporting belongings from one location to another. Through a combination of all these factors, the perfect mobility assumptions of Tiebout can clearly be seen as inaccurate. When these costs are included in the framework, the perfect “price-taking” nature of the firm-city deteriorates to some degree. Demand for a residential location ceases to be perfectly elastic as consumer-voters are forced to compare the benefits of an improved LPG package with the costs associated with moving to attain it.

**Employment Concerns**

The reality of the assumption of locational decisions being independent of employment concerns and their geography, while useful for modeling purposes, has similar impacts on the accuracy of a model’s results. While employment concerns do not affect some individuals, the majority of the population does face this factor. When considering those individuals whose jobs require their presence at a fixed geographic location, the change in the costs of commuting become relevant when addressing any relocation decision (Flatters, Henderson, & Mieszkowski, 1974; Thiss & Wildasin, 1992; Wheaton, 1977). To do otherwise would bias their individual utility estimates just as surely as the absence of geographical considerations undercut the nonspatial club or fiscal federalism theories. A move that dramatically changed commuting costs could easily outstrip any gains or losses from governmental efficiency and LPG package attributes.

**Residential Opportunity**

Another unrealistic aspect in the hypothetical construction of the Tiebout model is the implicit assumption that all individuals are equally able to acquire housing and reside in all communities. Numerous studies have shown that there are great differentials in housing prices between communities within the same metropolitan area, as well as between metropolitan areas. Only those individuals with the financial resources to afford housing in the most expensive of communities are afforded this privilege. Those with lower incomes or wealth must make their selections from the constrained set of choices that they can afford. The lower the income of an individual or a household, the smaller that set of possible housing locations will be. In addition, there are frequently nonfinancial barriers that restrict the access of certain individuals to housing in certain areas. Whether the grounds for exclusion are racial or class oriented, tools such as realtor steering, zoning restrictions, household composition regulations, neighborhood and homeowners’ associ-
ations, and social pressure are all used to keep unwanted individuals out of certain communities, regardless of their financial capabilities (Darden, Hill, Thomas, & Thomas, 1987; Garreau, 1991; Hirsch, 1983; Massey & Denton, 1993; McGuire, 1974; Stein, 1995; Wilson, 1987).

**Additions to the Tiebout Framework**

In addition to the restrictions imposed by the basic assumptions of the Tiebout model, other significant factors influence individuals’ choices of residential locations and LPG packages. Two of these factors, strongly suggested by the preceding discussions of the shortcomings of the Tiebout assumptions and other works in the urban literature, are considered here as additions to a Tiebout-type framework. These additions are by no means an exhaustive list of potential changes, but they provide a powerful first step toward a more useful and widely applicable tool of analysis. When combined with the changes suggested by the discussion in the previous section, these factors provide the basis for this article’s model.

**Economic Factors**

As implied by the discussion above, the cumulative influence of indirect and direct costs of a change in residential location presents a major potential influence on the locational decisions of individuals regardless of the relative LPG properties of the communities considered. As a result, the revised model will include a consideration of the information, transaction, and differential commuting costs that are present in every potential relocation. Even if we accept the maximization of personal utility through LPG selection as the ultimate goal of locational decisions, ignoring the influence of these other economic variables leads to a false picture. Indeed, with their inclusion, one sees not a purely competitive marketplace with all individuals seeking out the theoretically “best” LPG package, but rather a process of balancing and compromising. Specifically, we should not expect a rational individual to move merely because another location or community offers a package of LPGs that maintains or increases the consumer-voter’s utility at equal or lower tax cost. Instead, we should expect individuals to move only when the marginal long-run benefit (or discounted future value) of a competing LPG option exceeds the costs associated with moving to attain this package. In essence, this will result not in consumers seeking only the best quality, but rather the greatest value for every dollar. Indeed, a move could even be expected to occur in cases where the cost of the LPG package increased as long as the marginal cost (of both the LPGs and the move itself) was exceeded by the marginal value either of the new service package or savings in other areas of a consumer’s life (such as commuting).

A direct implication of this reduced propensity of consumer-voters to move in search of LPG values is the consequences for changes in government action are less drastic than implied by the initial Tiebout assumptions. A change in the local LPG package away from the ideal of an existing resident is not guaranteed to drive that consumer-voter to move because of the costs involved in switching locations. This slack in resident response may provide the freedom for governments to attempt to increase the attractiveness of their community to outsiders in order to improve economies of scale, but it will hinder efforts to reduce populations for the same reason.
The Role of Race

Another issue ignored in the formulation of Tiebout models, but one that has played a major role in the development and history of the residential environment of American metropolitan areas is race. Rather than being a non-factor, for most of the history of the US, race—particularly the black-white dichotomy—has been an absolute determinant in both the quality and the location of urban residential opportunities. Even today, extreme segregation exists in most major urban areas. Some of the inequalities in residential opportunity mentioned above may be attributed to behavior of this type. While legal enforcement of segregation is no longer permitted, there is strong evidence that suggests that for some portion of the American public, the racial balance of residential areas and public facilities remains a significant influence on the perceived attractiveness of the location. As such, the racial composition of communities should not be ignored in the calculus of residential location (Darden et al., 1987; Hirsch, 1983; Massey & Denton, 1993; McGuire, 1974; Vandell, 1995).

AN EXTENDED MODEL AND NEW IMPLICATIONS

In accordance with the factors listed above, the basic Tiebout-style model and its analysis should be expanded to include measures of the currently omitted forces impacting the decisions of individuals to relocate from one location to another. One of the major forces transforming the landscape of locational choice in metropolitan areas is the rapid growth of privatization of traditionally public services in one form or another (Garreau, 1991; Kettl, 1993; Lemov, 1994; Mahtesian, 1994). Whether such privatization includes contracting for police services from a nearby jurisdiction or the completely self-contained maintenance of streets, utility lines, and security services of a gated community, these nontraditional models of service provision are of growing importance in the study of urban areas, particularly in areas of new growth and development. This type of service provision and related extensions to the basic Tiebout framework will be specifically targeted here.

The Pseudo-Public Goods Approach

Through the development of a formal game theoretic model analyzing the decisions of residents, developers, and government with respect to adopting a specific form of privatized LPG, it will be possible to derive an additional set of manifestations that allow the accuracy and predictive power of the enhanced Tiebout-type model to be tested in a new, modern context. Related to the economic factors cited as shortcomings, we would expect citizens who have not moved but are not receiving the most valued set of LPGs afforded by their economic situation to seek some or all of the absent goods without the costs related to moving (Gross, 1995). In addition, it is rational to expect communities, especially in a climate of privatization and limited government, to seek extra-governmental mechanisms to facilitate the continued growth and development of their communities.

Similar to the voice and contracting hypotheses discussed in Lowery and Lyons (1989), these endeavors could take one of two forms depending on circumstances within the community. First, in a location where the group of residents is sufficiently large to constitute a political majority, the citizens should be expected to act through official governmental
channels. The advantages of an action of this sort lie in the financing options available. If an option is provided to a community in general by the government, its costs are spread across all residents regardless of their preferences for the good. (The effect may be nullified to some extent by the reliance of the government on user fees for a portion of the cost of the new good, but it will be present as long as some general revenue is used for the project.) This allows the agitating residents, in effect, to free ride to higher utility levels on the backs of their less concerned and motivated neighbors (Olson, 1965). This ability to exploit the lower interest levels of others is permitted by the greater inertia in mobility previously asserted.

In those situations in which a spatially concentrated group of individuals is interested in more services, but their numbers are not sufficient to initiate political action, we may see individuals pursuing a strategy resulting in a privatized analogue of an LPG—the pseudo-public good. These goods will be typified as special amenities tied to owning property or leasing housing within a specific development. They typically will be available only to residents of the development or neighborhood (or open to the public at a significantly higher cost) and will be funded through such mechanisms as homeowners’ association dues, or maintenance and recreation fees. The association through which the good is created serves as a “shadow government,” and the dues or fees (enforced with the power of law through a title, lease, purchase agreement, or other contract) serve as the privatized analogue of a tax (Garreau, 1991).

A strategy of enhancing local amenities through relations of this type may be preferred to the traditional governmental approach even in those cases in which a successful political coalition is possible. The privatized approach will allow the involved property owner a larger voice in the decisions governing the goods he or she enjoys (and the money paid to provide them) by shrinking the constituency involved in the “political” process from the entire community to the individual development. The private approach may also be a less costly alternative to public provision for a number of reasons, but primarily focusing on access. If a specific good becomes a priority of an entire municipality, access to that good usually will be expected by the entire populace. If the good does not display the nonrival characteristic of pure public goods, it then follows that any given per capita level of provision will be more costly for an entire community than for a single neighborhood or development. In those cases in which the new good is sought primarily by the more affluent population of a community (the group most likely to be able to afford the costs of increased amenities), their higher average incomes, consumption levels, and property values will leave this group likely to bear the brunt of the cost of new goods. Thus, by successfully limiting provision of the good to their subset of the population through privatized pseudo-governmental means, such a group is quite likely able to reduce their total costs for providing the good through reliance on privatized institutions.

Game Theoretic Model

The balancing of these scenarios is the process investigated by the model presented in this article. The model is a three-player, extensive form game with complete and perfect information. Nature plays the role of assigning values for a set of variables from which payoff levels for players are determined. The specific variables and their meanings are discussed below. For the sake of tractability, the information is assumed to be costless.
The players are also assumed to be rational, utility-maximizing actors. The structure of the game and the payoff structure are presented in Figure 1.

**Players**

The first player is Developer, defined here as the developer/management of a residential subdivision of substantial size. In this game, this player is considered to be producing a new development in a metropolitan area. The Developer is assumed to have the power to place restrictions and conditions on the purchase and control of property within the development. This power provides it or its management agent with direct, mandatory, and legal influence over all residential units within the development that can be used to force action on the part of the residents. This player could also be conceived of as a homeowners’ association or any similar group in which a property owner is forced to participate. In producing the development, the Developer may seek to differentiate the project from the status quo of the community by including an LPG not currently offered in the area. This good may either be built by the Developer but run by the City as a traditional public good or be built and run by the Developer as a privatized pseudo-public good.
The second player is the City, which is defined as the local governmental unit (e.g., city, town or township) responsible for the zoning and approval of building and development plans and the existing populace that makes up this unit. The governmental unit is assumed to act in the interests of its existing constituency, and it will make its decisions on development projects on the basis of these interests. For the purposes of this game, it is assumed that the existing housing stock of the region is of uniform quality and price and the region’s racial composition is homogeneous.

The final player is the Consumer-Voter (C-V). This player represents the individual seeking to make a decision about purchasing into the proposed development in order to receive the differential benefits associated with the LPG package offered by the development.

**Modeling Payoffs**

The payoffs of this game are modeled in such a way that they are completely determined by a set of five variables: $d$, $a$, $V_I$, $t_{SQ}$, and $r_c$, where $d$ is the size of the development as a percentage of the existing city population; $a$ is ratio of housing value and tax base in the development to the city; $V_I$ is the value of a pseudo-public good enhancement to the LPG package to an individual household; $t_{SQ}$ is the status quo property tax rate in the city (which is assumed to be the only local source of revenue); and $r_c$ is the racial minority population share of the existing city. These variables represent the information required by the players in order for the complete and full information assumption to be met.

In this model, the proposed amenity enhancement to the development in question, a generic improvement to the LPG package, will be considered in marginal terms, thus allowing the opportunity cost to the C-V of purchasing into the development (and giving up his or her existing LPG package) to be set at zero. This assumes that the basic LPG package of the new location, absent any improvements, is equally preferred to the C-V’s existing package—likely a conservative assumption considering costs facing a potential mover as mentioned above. As such, if the net value of the amenity improvement (benefits of the good minus associated costs) is positive, a move will be favored; if it is negative, no move will occur, and is zero the C-V will be indifferent. In the public case (payoff $C-V$) the payoff consists of a weighted measure of the value ($V_I$) of the amenity enhancement to an individual household minus its share of the tax increase ($Cost_{tax}$) necessary to pay for the LPG package enhancement and any perceived costs resulting from a change in the racial mixture of the neighborhood tied to the production of the good ($R_O$).

In the case of public provision, the household value ($V_I$) is weighted to a value less than one because the publicly provided amenity is open for use by the entire city, not just residents of the development. Because the good is located within the development, however, the largest share of the value is assumed to be captured by its residents because of their proximity. The racial effect term ($R_O$) is included in response to the market stratification arguments about the traditional form of Tiebout models made above. Specifically, this term will capture any changes in market value of the property induced by perceptions of any differential racial composition existing within the development. It is important to remember that the racial mixture of the development need not match that of the larger region. The calculation of $r_d$, the racial minority population share of the de-
velopment, takes relative housing prices into account. As housing prices rise, the share of minority population falls. This captures the effects of economic and class segregation (and the related racial effects) present in most housing markets (Massey & Denton, 1993).

In the pseudo-public case (payoff $C-V'$), the costs of the amenity are borne completely by the residents of the development. As such, the cost and value terms drop out of the payoff equation leaving only the perceived racial impact term ($R_F$).

The value to the City of any new amenity is the net decrease in tax rates produced for existing residents, which results from the construction of the new development with the enhanced LPG package, as well as any direct benefits received from the good itself. In the case of public provision (payoff $City$), spillover benefits from the good are present. In this model, the cost of existing city services are assumed to grow in a linear manner as the number of households increases. As such, if the price of housing in the new development exceeds that of the existing housing stock, revenue will increase at a faster rate than costs with a constant tax rate. This mechanism drives the potential benefits described above. As in the case of the C-V, the individual household in the city again receives a weighted share of the value of the good. This share is less than that included for the Consumer-Voter because the location of the amenity within the development decreases its relative accessibility despite its public nature. This payoff also includes the two potential changes in taxes faced by the existing homeowners—any increases necessary to pay for the public good ($Cost_{tax}$) and any changes in property taxes for general services produced by differential housing costs ($\Delta T_C$). In the pseudo-public case (payoff $City'$), complete private provision and the related private access result in all direct benefits and costs related to the good going to zero. All that remains is the term assessing the change in property taxes ($\Delta T_C$).

The value to the Developer will be any increased profits from the sale of property in the development resulting from LPG package enhancements and any other related costs or benefits. The increased profits result from the perceived increases in the value of the housing due to the additional amenity. For the Developer, these costs are aggregated across the entire development. As such, the Developer payoffs closely mirror the benefit portions of the C-V payoffs. In the case of public provision (payoff $Dev$), this is the development-captured share of the weighted good benefits. This share is captured by multiplying the $V_I$ term by the size of the development ($S_D$), resulting in the $V_T$ term (and the same weighting factor). From this value are subtracted the aggregated perceived costs of racial composition changes, which are again calculated by multiplying the individual C-V value by the size of the development ($R_F \cdot S_D$). The construction of the payoff for the pseudo-public case ($Dev'$) is highly similar. The only major change is that the weighting factor is absent from the equation, leaving only the aggregated value of the good for the development ($V_T$) and the perceived racial costs ($R_F \cdot S_D$). The exact definitions of all these payoffs and their derivations from the five basic variables are presented in Table 1.

**Analysis of Game**

As this is a game of complete and perfect information, equilibrium conditions may be derived from a simple backwards reduction. At each decision node, the C-V will opt to purchase into a development only if its benefits are at least as great as its costs. Thus, if $Buy$ or $Buy'$ are greater than or equal to zero, C-Vs will be willing to purchase at that node and the development will proceed. If the payoffs for $Buy$ or $Buy'$ are negative at either of
Table 1

Definitions of payoffs at (Public, Approve, Buy):

\[
\begin{align*}
Dev &= \frac{1}{1 + \delta} (V_I) - (R_R \cdot S_D) \\
City &= \frac{\delta}{1 + \delta} (V_I) - \Delta T_C - Cost_{tax} \\
C - V &= \frac{1}{1 + \delta} (V_I) - Cost_{tax} - R_O 
\end{align*}
\]

Definitions of payoffs at (Pseudo-Public, Approve, Buy):

\[
\begin{align*}
Dev &= V_I - (R_R \cdot S_D) \\
City &= -T_C \\
C - V &= V_I - Cost_{fees} - R_R = -R_R 
\end{align*}
\]

Definitions of variables:

- Size of City = S_c = 10000 households
- Size of development = S_d = S_c where 0.01 ≤ S_d ≤ 0.20
- Value of existing house (and tax base) = H_e = 100000
- Value of development housing (and tax base) = H_d = H_e where 0.50 ≤ H_d ≤ 2.00
- Value of pseudo-public LPG per Development household = V_I where 1000 ≤ V_I ≤ 10000
- Total value of LPG = V_T = S_d \cdot V_I
- Cost/City household of public good via tax = Cost_{tax} = \frac{V_T}{(S_c \cdot H_e) + (S_d \cdot H_d)} \cdot H_e
- Cost/Devel household of public good via tax = Cost_{tax} = \frac{V_T}{(S_c \cdot H_e) + (S_d \cdot H_d)} \cdot H_d
- Cost/Devel household of pseudo-public good via fees = Cost_{fees} = \frac{V_T}{S_d} = V_I

These tax-financing measures calculate a property tax rate sufficient to meet the cost of provision for the public good enhancement. The rate is determined by dividing the total value of the good by the total property value of the city. Costs for city and development residents are then obtained by multiplying the derived good-related tax rate by the respective housing values of the two groups.

Given an initial City tax rate \( s_0 \), where 0.01 ≤ s_0 ≤ 0.05 (equal to a property tax rate of between 10 and 50 mils)
- Initial budget = B_{SO} = s_0 \cdot H_e \cdot S_c
- New budget with development = B_0 = (1 + \lambda B_{SO}
New tax rate with development $= \tau_D = \frac{B_D}{H_S + H_d S_d}$

Change in City taxes $= \Delta T = (\tau_D - H_S) - (\tau_0 - H_S)$

These equations assume that the costs of existing city services (or LPGs) increase in a linear manner as population grows. Thus, each additional household costs the same amount to serve as each existing household. Any additional necessary infrastructure, such as sewers, school buildings, and the like are assumed to be constructed by the developer with costs covered by the price/value of the new homes.

Benefits/City household of public goods $= \left( -\frac{\delta}{1 + \delta} \right) V_i$

Benefits/Development household of public goods $= \left( \frac{1}{1 + \delta} \right) V_i$

Benefits from the publicly provided LPG enhancement are assumed to spill over to the community at large at some rate determined by the ratio of the size of the development and the existing city. The smaller the development is compared to the size of the city, the more completely the benefits are captured by the residents of the development. The differential benefit rates are based on the effects of awareness of goods, and perceived likelihoods of congestion (for goods where congestion is applicable). Different weighting schemes are possible; however, the general effects will be similar.

Racial minority pop. share of City $= \rho_c$ where $0.01 \leq \rho_c \leq 0.30$

Racial minority pop. share of Development $= \rho_d$

Cost of change in racial mixture with open public good $= R_O$, where

$$R_O = H_d - \frac{H_d}{e^{\Delta \text{mix}_1}}$$

$$\Delta \text{mix}_1 = \left[ \left( \frac{\rho_c S_c + \rho_d S_d}{S_c + S_d} \right) - 0.1 \right] - |\rho_d - 0.1|$$

Cost of change in racial mixture with restricted pseudo-public good $= R_R$, where

$$R_R = H_d - \frac{H_d}{e^{\Delta \text{mix}_2}}$$

$$\Delta \text{mix}_2 = \left[ |\rho_d - 0.1| - |\rho_c - 0.1| \right]$$

The R terms for costs/benefits associated with changes in racial balance are symmetric about the chosen ideal point, here 10%. Each reflects these costs or benefits as a share of the housing value gained or lost as the chosen LPG provision paradigm (public or pseudo-public) is compared to a base case.

Price of home in development equals value of home (tax base, pure market value of structure and land) plus value of associated goods enhancements minus racial costs, or

Price with open access public goods $= P_O = H_d + \frac{1}{1 + \delta} (V_i) - R_O$

Price with restricted access pseudo-public goods $= P_R = H_d + V_i - R_R$
these nodes, C-V will choose *Not-Buy* or *Not-Buy’*, the project will be vetoed by a lack of interest, and all players will receive a payoff of zero.

It is worth noting that, in this model, it will be assumed that players take affirmative action at a point of indifference in their payoffs. Thus, when indifferent between purchasing or not purchasing into a development, the C-V will choose to purchase. Likewise, City and Developer will opt to proceed with the development plans rather than veto at points of indifference. Similarly, when Developer is indifferent between public and pseudo-public goods, we will assume that the public strategy is chosen. While this differs from the standard game theoretic approach, which prescribes a mixed strategy at such “knife-edge” points, these situations are sufficiently rare in the real world that ignoring their analysis detracts little from the game while greatly simplifying the analysis.

Moving back to the decision nodes facing City, we face a slightly different situation. At this point, City will be able to foresee the actions of C-V. If C-V is willing to purchase, City bases its decision on the payoff associated with the project going through (*City* in the public case or *City’* in the pseudo-public case). If in any case the City payoff resulting from C-V’s subsequent choice is negative, City will choose *Not Approve* or *Not Approve’*, the project will be vetoed, and all players will receive zero. If in any case the payoff is zero or greater, City will approve. If, however, City knows that C-V will choose the Not-Buy option available at the next decision node, City’s choice between approving and not approving the project as each will be irrelevant in determining the final outcome of zero for all, but City will still act in a manner consistent with utility maximization and will choose the path recommended by its analysis.

Developer’s actions are relatively easy to describe. If both of the development paths available to it (*Public* or *Pseudo-Public*) lead to negative payoffs, it will put forward no development proposal, and all parties will receive zero. If, following the backwards reduction process, one or both of these paths lead to a non-negative payoff, Developer will choose whichever path produces the greater expected value. In situations in which two or more paths result in the same maximum value, Developer will proceed in the manner described above.

Given this information, we can make the following statements about the equilibria of this game. First, the exact equilibrium of any game is dependent on the values assigned to the variables $\delta$, $\alpha$, $V_I$, $\tau_{SQ}$, and $\rho_\ldots$. It is important to note that the value of 0.1 used in the calculation of $R_D$ and $R_R$ may also be considered a variable of this type. This quantity represents the ideal or utility maximizing level of racial integration for a community. This particular value is suggested by Massey and Denton (1993), but other values could be used. This topic will be revised below. In this model, the assignment of the five basic variables is made randomly from within a set of reasonable ranges listed in Table 1, by Nature (designated N in the game tree). Second, because of the restriction on knife-edge points enunciated earlier, only sub-game perfect Nash equilibria without mixes will exist for any set of starting values assigned to the variables. Depending on the specific values taken by these five variables, five potential scenarios may result. These cases are summarized in Table 2.

Due to the multiple veto points present in the development process, a development must clear a very high hurdle before it is undertaken. Because each player has veto power over the process and may nullify any plan that is harmful to his or her interests, only improvements which are Pareto superior may occur. In addition, it is clear that by moving
TABLE 2

CASE 1: NO OPTIONS FEASIBLE, NO DEVELOPMENT OCCURS

Because of the veto points for each player with regard to each development option, for any development option to succeed with the starting values assigned by Nature, each player must have a non-negative payoff associated with that option. If each LPG production option (Public or Pseudo-Private) is associated with a negative payoff for at least one player, then no development will occur. Any combination of strategies which contains at least one of No Proposal, Not Approve, Not Buy, Not Approve, or Not Buy can be an equilibrium for this case. The case may be stated more formally as:

If: \( \text{Dev} < 0 \)
and/or \( \text{City} < 0 \)
and/or \( C - V < 0 \)
And: \( \text{Dev} < 0 \)
and/or \( \text{City} < 0 \)
and/or \( C - V < 0 \)
Then: no development will occur

Payoffs: (0,0,0)

CASE 2: ONLY PUBLIC LPG PROVISION IS FEASIBLE AND OCCURS

If, for a given set of variable assigned by Nature, at least one payoff associated with the Pseudo-Public option is negative, while all payoffs associated with Public are non-negative, the Public LPG option will occur. The resulting equilibrium is (Public, Approve, Buy). A formal definition of the case is:

If: \( \text{Dev} 0 \)
and \( \text{City} 0 \)
and \( C - V 0 \)
And: \( \text{Dev} < 0 \)
and/or \( \text{City} < 0 \)
and/or \( C - V < 0 \)
Then: Public development will occur

Payoffs: (Dev,City,C-V)

CASE 3: ONLY PSEUDO-PUBLIC LPG PROVISION IS FEASIBLE AND OCCURS

If, for a given set of variables assigned by Nature, at least one payoff associated with Public is negative, while all payoffs associated with Pseudo-Public are non-negative, the Pseudo-Public LPG option will occur. The resulting equilibrium will be (Pseudo-Public, Approve, Buy). A formal definition of this case is:
CASE 4: BOTH LPG PROVISIONS FEASIBLE, PUBLIC OCCURS
If, for a given set of variable assigned by Nature, all payoffs associated with the Pseudo-Public option are non-negative, while all payoffs associated with Public are also non-negative, and Developer has a payoff from Public greater than or equal to that from Pseudo-Public, the Public LPG option will occur. The resulting equilibrium will be (Public, Approve, Buy). A formal definition of this case is:

If: $Dev < 0$
and/or $City < 0$
and/or $C - V < 0$
And: $Dev' \geq 0$
and $City' \geq 0$
and $C - V' \geq 0$

Then: Pseudo-Public development will occur
Payoffs: $(Dev, City, C-V')$

CASE 5: BOTH LPG PROVISIONS FEASIBLE, PSEUDO-PUBLIC OCCURS
If, for a given set of variable assigned by Nature, all payoffs associated with the Pseudo-Public option are non-negative, while all payoffs associated with Public are also non-negative, and Developer has a payoff from Pseudo-Public greater than that from Public, the Pseudo-Public LPG option will occur. The resulting equilibrium will be (Pseudo-Public, Approve', Buy'). A formal definition of this case is:

If: $Dev \geq Dev'$
and $City \geq 0$
and $C - V \geq 0$
And: $Dev' > 0$
and/or $City' \geq 0$
and/or $C - V' \geq 0$

Then: Pseudo-Public development will occur
Payoffs: $(Dev', City', C-V')$
first, the Developer has a more powerful role than the other players in the game. The Developer, facing a situation in which both the Pseudo-Public or Public options will meet with the approval of both City and C-V, has the power to force his preferred outcome upon the other players. Let us now consider some examples that demonstrate the impact of the starting values of the variables on the expected outcomes.

Two simulated potential environments for development are produced. In each of these areas, four of the five variables that contribute to the payoffs of the players are held constant, while the fifth is allowed to vary over a representative range. The fixed values are $\delta = 0.10$, $\alpha = 1.2$, $V_I = 5000$, $\tau_{SO} = 0.03$ (or 30 mils), and $\rho_c = 0.20$. These values are all chosen, except in the case of $\rho_c$, as relatively central values in the theorized ranges considered in Table 1. Figures 2 and 3 depict the payoff values which result when one of these variables is varied across its range, while the rest are held constant. It is important to note one difference between these curves and the expressions from which the payoffs are derived. In order to feasibly display all payoffs on a single graph, the payoffs displayed in the graphs for City and C-V have been aggregated to a collective level, whereas the expressions in Table 1 are defined at the individual level. The conversions are made by multiplying the City payoff produced by the expression in Table 1 by the size of the city ($S_C$) and, likewise, by multiplying the C-V payoff by the size of the development ($\delta \cdot S_C$). No sign changes or shifts in feasible ranges of data are produced by these changes. They simply serve to increase the magnitude of the resulting figures to a range where they are able to be displayed on the same metric as Developer payoffs.

In Figure 2, $\delta$, the size of the development relative to the size of the existing city, is varied between 0.01 and 0.20. Note that in this case, both public and privatized goods provision are feasible over nearly the entire range of the analysis, given the values assigned to the other variables. It is not until $\delta$ reaches approximately 0.18 that the payoffs for C-Vs become negative on the Public option. Even through this range, however, all three payoffs for the Pseudo-Public goods option remain non-negative. This means that the privatized option will be pursued for all values of $\delta$ above the point where the C-V payoff becomes negative. As for values of $\delta$ below this threshold, we know from the analysis in Figure 2 that when all payoffs are positive, the option pursued will be that which results in the largest payoff for Developer. In this case, the line representing $Dev'$ lies well above that of $Dev$ for all levels of $\delta$. As such, the Pseudo-Public option will again be the equilibrium result.

Figure 3 shows the results produced by varying $\rho_c$ between 0.01 and 0.30 (or varying the minority composition of the community between 1% and 30%). As a result of the absolute value terms used in constructing the $R$ values feeding into the payoffs for Developer and C-V, the curves for these players have sharp angles rather than the gentle curves of Figure 2. By observing the general direction of the Public option payoffs for these two players as compared to those of the Pseudo-Public option, we see the differing impacts of metropolitan racial composition on the two provision paradigms. As the minority population grows beyond a critical value at approximately 0.11, so does the advantage of the privatized option. This is because this option more forcefully segregates the development from the surrounding city than does the public approach. The more the municipal minority population share exceeds the socially defined “ideal” level discussed above, the more that isolation is prized.

The analysis of this figure provides the intuition behind one of the more interesting potential uses for this type of analysis. As noted above, the analyses presented here as-
FIGURE 2
Payoffs by Values of Delta
FIGURE 3
Payoffs by Levels of rhoC
sume a homogeneous set of preferences among the population, which includes attitudes toward ideal levels of racial integration. The C-V results produced here assume that both majority and minority individuals have a mixture of approximately 90% majority/10% minority as the ideal. This assumption, while very convenient for producing models, may not ring true for all groups in society. The same literature that suggests that majority citizens possess an ideal point in the range of 10% for the proportion of minority population also suggests a much different ideal point held by minority groups, one in the range of 50%–60% (Massey & Denton, 1993, pp. 88-96). Were the utility function for C-V’s split along racial lines to reflect these differing preferences for levels of racial mixing, similar to that suggested by Massey and Denton (1993), two C-V payoff lines—one for majority households and one for minority households—would then result. Figure 4 recreates an analysis of this type.

In this figure, C-V and C-V′ for prospective minority movers is calculated with an ideal point of 0.3, which replaces the value of 0.1 in the $\Delta_{mix_1}$ and $\Delta_{mix_2}$ equations listed in the Table 1, and transforms them into the expressions found in Table 3. In addition, the value of $\rho_c$ is allowed to range from 0.01 to 0.59 (corresponding to a minority population of 1% to 59% in the existing city) in increments of 0.02 (or 2%).

One valuable piece of information that this analysis reveals is the point at which the two (majority and minority) C-V payoff curves and C-V′ payoff curves are close or equal, such as the two C-V curves in the range where $\rho_c$ is below approximately 0.09. These points represent combinations of the key variables where each group has equal or nearly equal values for the development’s housing when considering the racial mixture of the area. This point represents a stable equilibrium at which an integrated neighborhood could theoretically be maintained. As variables change and move away from points of that type, the development or neighborhood may begin to “tip” or be more strongly biased toward segregation rather than integration. This type of situation begins to present itself as we observe $\rho_c$ rising above the 0.09 threshold. In this range, through a $\rho_c$ value of approximately 0.34, the neighborhood would tend toward greater minority segregation, as the value of the housing as perceived by minority consumer-voters diverges in a positive direction from the majority’s perceived value. Even in ranges such as $\rho_c = 0.17$, increasing segregation could be expected as Developer could extract a far larger premium by increasing minority sales. Obviously, the conditions necessary for maintaining a sustained integrated neighborhood will not be as highly sensitive as this model implies, for the residential inertia reasons discussed above, but this type of analysis can help demonstrate how highly segregated residential areas have remained predominant despite increased racial tolerance in society. Only a small difference in racial ideal points is enough to drive the C-V and C-V′ curves apart and open the doors to the possibility of conditions ripe for turnover.

**CONCLUSIONS**

These simulations suggest that we should expect to see a prevalence of the traditional public strategy and thus a higher level of government-provided LPGs in those communities that are homogeneous in both race and housing values. As the homogeneity of these interrelated measures begins to break down, we should expect levels of publicly provided LPGs to drop and pseudo-public provision to grow. As the degree of differentiation in
race grows between a development and the surrounding community, the greater the steps it will take to move its composition and racial integration (either through population or LPG use) closer to the socially defined ideal level. In more general terms, we would expect privatized pseudo-public good provision to be positively correlated with neighborhood (or development) income and property values levels in high-minority areas. It seems reasonable to predict that the privatized pseudo-public goods development strategy will become increasingly common as central cities attempt to redevelop and expand their residential base. Limited access services of this type may be used as tools to lure back the previously fleeing middle class by providing spatial niches with suburban-style amenities and more preferred racial compositions within the urban core.

Models of this type also provide strong opportunities for quantification and empirical study. By combining a formal model of this type or a more extended version with commonly available data, such as census demographics and studies of the US housing stock, it may be possible to more systematically isolate the variables driving current residential growth and locational trends. Analyses of this type, combined with further elements of the extended Tiebout-type model presented above, may be extended to the level of the development or subdivision (as a proxy for neighborhoods) or census tract. The recent emergence of tools for attacking the ecological inference problem (King, 1997) provide yet another tool for strengthening assessments of the significance and import of formalized Tiebout-type models, as do ever-improving geographic information systems and spatial econometric methods (Anselin, 1996). The combination of these techniques in future research should shed new light on the dynamics of metropolitan housing development, public good provision, and neighborhood integration and racial turnover.

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REFERENCES


