Strategic Interaction Among Governments

by

Jan K. Brueckner
Department of Economics
and
Institute of Government and Public Affairs
University of Illinois at Urbana-Champaign
1206 South Sixth St.
Champaign, IL 61820
e-mail: jbrueckn@uiuc.edu

June 2003
1. Introduction

Strategic interaction among governments has recently become a major focus of theoretical and empirical work in public economics. One branch of the literature analyzes strategic interaction due to various kinds of “spillovers.” For example, in a model with pollution spillovers, the preferred level of abatement in one jurisdiction depends on policies chosen elsewhere (see Wilson (1996) for a survey). While other types of spillovers also lead to interaction, a different mechanism is at work in tax-competition models, where governments levy taxes on a mobile tax base. Recognizing that their tax bases shrink as tax rates rise, large jurisdictions choose their rates in strategic fashion, with an eye on choices made elsewhere (see Wilson (1999) for a survey). A related literature focuses on “welfare competition,” analyzing income redistribution by state governments when the poor migrate in response to differentials in welfare benefits. In such models, states choose benefit levels in strategic fashion taking account of the mobility of the poor and the choices of other jurisdictions (see Brueckner (2000) for a survey).

Spurred in part by these theoretical developments, strategic interaction among governments is now the focus of a growing empirical literature. Most studies in this literature test for strategic interaction by estimating reaction functions, which show how a jurisdiction responds to the choices of neighboring jurisdictions in setting the level of its own decision variable. If the estimated reaction function shows interdependence among policy choices, the presence of strategic interaction is confirmed.

The purpose of this chapter is to provide an overview of conceptual issues in the theoretical and empirical literatures on strategic interaction. The discussion begins by showing how most theoretical models of interaction fall into one of two main categories: spillover models such as the pollution model discussed above, and “resource-flow” models, a category that embraces both tax- and welfare-competition models. The discussion in Section 2 provides a general
characterization of each type of model, while sections 3 and 4 provide detailed analysis of particular examples. To illustrate the spillover model, section 3 analyzes a model of strategic interaction in the choice of pollution abatement levels. To illustrate the resource-flow model, section 4 provides a detailed analysis of the welfare-competition model.

Section 5 explains the main econometric issues that arise in testing for strategic interaction. A fundamental problem in estimating jurisdictional reaction functions is the endogeneity of key covariates. In particular, the decision variables of other jurisdictions, which appear on the right-hand side of the regression, are jointly determined along with dependent variable in a Nash equilibrium. As a result, ordinary-least-squares estimates of the reaction-function coefficients are inconsistent, so that a different estimation method must be used. Another problem concerns the potential for the emergence of false evidence of strategic interaction. When unobservable determinants of policy choices are correlated across jurisdictions, this correlation can generate a spurious association between policy choices that may be mistaken for strategic interaction. The discussion outlines methods for dealing with this problem, which is known as spatial error dependence.

2. A Typology of Models with Strategic Interaction

Models of strategic interaction among governments can be classified into two broad types: the “spillover” model and the “resource-flow” model. Despite their different structures, these models ultimately generate a same kind of behavioral relationship, a jurisdictional reaction function, which can be estimated empirically. The subsequent discussion provides a general characterization of the spillover and resource-flow models, and sections 3 and 4 provide concrete examples.

2.1. The spillover model

In the spillover model, each jurisdiction $i$ chooses the level of a decision variable $z_i$. However, the jurisdiction is also directly affected by the $z$’s chosen elsewhere, indicating the presence of spillovers. Thus, jurisdiction $i$’s objective function is written

$$V(z_i, z_{-i}; X_i),$$

(1)
where \( z_{-i} \) is the vector of \( z \)'s for the \( n-1 \) other jurisdictions and \( X_i \) is a vector of characteristics of \( i \), which help determine preferences.

Under Nash behavior, jurisdiction \( i \) chooses \( z_i \) to maximize (1), taking \( z_{-i} \) as parametric. The resulting first-order condition is \( \partial V / \partial z_i \equiv V_{z_i}(z_i, z_{-i}; X_i) = 0 \), and solving this equation for the optimal \( z_i \) yields a solution that depends on \( z_{-i} \) and \( X_i \). Thus, the optimal value of jurisdiction \( i \)'s decision variable depends on choices elsewhere and on \( i \)'s characteristics. The solution can be written

\[
z_i = R(z_{-i}; X_i).
\]

(2)

The function \( R \) represents a reaction function, which gives jurisdiction \( i \)'s best response to the choices of other jurisdictions. Note that the position of the reaction function depends on jurisdiction \( i \)'s characteristics.

Empirical work on strategic interaction focuses on estimating the slope of the reaction function, as explained in detail below. In the spillover model, this slope depends on the nature of preferences. Differentiation of the above first-order condition shows that \( \partial z_i / \partial z_j \equiv R_{z_j} = -V_{z_i z_j} / V_{z_i z_i} \), \( j \neq i \), where \( z_j \) is one of the elements of \( z_{-i} \) and the latter expressions are the second partial derivatives of \( V \). Although \( V_{z_i z_i} \) must be negative for the second-order condition to be satisfied, \( V_{z_i z_j} \) can take either sign, depending on the nature of preferences. If an increase in jurisdiction \( j \)'s \( z \) value raises the marginal utility of \( z_i \), then \( V_{z_i z_j} \) is positive, and the reaction function is an upward-sloping function of \( z_j \). Conversely, if an increase in \( z_j \) decreases the marginal utility of \( z_i \), then the reaction function is downward sloping in \( z_j \).

The reaction function's slope could also be zero, an outcome that arises when preferences are additively separable in \( z_i \) and \( z_j \), implying \( V_{z_i z_j} \equiv 0 \). As a result, a zero estimated slope for an empirical reaction function does not necessarily imply the absence of strategic behavior among jurisdictions. However, since a zero slope is theoretically “unlikely,” such an empirical finding is probably best viewed as disconfirmation of the underlying model, indicating either that spillovers are absent or that jurisdictions do not take them into account in their decisions.

The Nash equilibrium in the spillover model is determined by the intersection of the jurisdictional reaction functions, which corresponds to the solution of the equation system con-
sisting of (2) for \( i = 1, \ldots, n \). If jurisdictions are symmetric, with \( X_i = X \) for all \( i \), then the equilibrium is also symmetric, with a common \( z \) value across all jurisdictions. This value is determined by the single equation \( V_{z_i}(z, z; X) = 0 \), where the partial derivative is understood to refer to \( V \)’s first argument, and where the second argument represents an \( n - 1 \) vector with each element equal to \( z \). Equivalently, using the reaction function, the equilibrium satisfies \( z = R(z, X) \), where the \( z \) on the RHS is again an \( n - 1 \) vector.

In the spillover model, Nash equilibria are inefficient, failing to maximize social welfare. This conclusion is most easily seen in the symmetric case, where the common socially optimal level of \( z \) maximizes \( V(z, z; X) \), satisfying \( V_{z_i} + \sum_{j \neq i} V_{z_j} = 0 \). At the Nash equilibrium, the first term in this expression is zero, while the summation in positive, assuming positive spillovers. As a result, the entire expression is positive at the Nash equilibrium, indicating that social welfare is still increasing in \( z \) and that equilibrium value is too low relative to the social optimum. The problem, of course, is that each jurisdiction does not take into account the external benefits that result from an increase in its \( z \) value. The conclusion is reversed if spillovers are negative, with the equilibrium \( z \) too large.

2.2. The resource-flow model

In the “resource-flow” model, a jurisdiction is not directly affected by the \( z \) levels in other jurisdictions. But the jurisdiction cares about the amount of a particular “resource” that resides within its borders. Because the distribution of this resource among jurisdictions depends on the \( z \) choices of all, jurisdiction \( i \) is then indirectly affected by \( z_{-i} \).

Jurisdiction \( i \)’s objective function in the resource-flow model is written

\[
\tilde{V}(z_i, s_i; X_i),
\]

where \( s_i \) is the resource level enjoyed by \( i \). The distribution of resources depends on the entire \( z \) vector as well as on jurisdiction characteristics, with the resources available to \( i \) given by

\[
s_i = H(z_i, z_{-i}; X_i).
\]
Note that since $X_i$ can be measured relative to the average characteristics of all jurisdictions, $X_{-i}$ need not appear in (4).

To derive the reduced form of the resource-flow model, (4) is substituted into (3), yielding

$$\tilde{V}(z_i, H(z_i, z_{-i}; X_i); X_i) \equiv V(z_i, z_{-i}; X_i).$$

(5)

Thus, even though the underlying model is different, this objective function has the same form as (1), with $z_i$, $z_{-i}$, and $X_i$ appearing as arguments. As a result, maximizing (5) by choice of $z_i$ yields a reaction function like (2).

The slope of the reaction function, which is again ambiguous in sign, now depends jointly on the derivatives of the $H$ and $\tilde{V}$ functions. Since this greater complexity means that the slope can be zero only under unusual conditions, a zero slope for an empirical reaction function is strong evidence against the underlying model.

As in the spillover model, the Nash equilibrium in the resource-flow model is inefficient. However, because of the greater complexity of the model, it is not possible to state simple, primitive conditions (e.g., spillovers positive vs. negative) that determine the direction of the inefficiency. Whether the decision variables are set at too high or too low a level depends on the details of the model’s structure.

3. A Spillover Example: Pollution Abatement

3.1. Model

To illustrate the spillover model, consider the case of pollution abatement, following Murdoch, Sandler and Sargent (1997) and Fredriksson and Millimet (2002). Let preferences for jurisdiction $i$ be given by $U(c_i, P)$, where $c_i$ is consumption and $P$ is the level of air pollution, which is assumed to be uniform across all jurisdictions. Suppose that pollution depends on the total abatement expenditures of all jurisdictions, with $P = P(\sum_{j}a_j)$, where $a_j$ is abatement expenditure in jurisdiction $j$ and $P^i < 0$. Letting $y_i$ denote income in $i$, it follows that $c_i = y_i - a_i$, assuming for simplicity that each jurisdiction has a single resident.
Substituting in $U$, preferences can be written

$$U(y_i - a_i, P(\sum_j a_j)) \equiv V(a_i, a_{-i}; y_i). \quad (6)$$

The first-order condition for choice of $a_i$ is $V_{a_i} \equiv -U_{c_i} + U_P P' = 0$. Assuming for simplicity that the $P$ function can be written in the linear form $\tau - \gamma \sum_j a_j$, so that $P' = -\gamma < 0$, the previous condition can be rewritten as

$$-\frac{\gamma U_P}{U_{c_i}} = 1. \quad (7)$$

This equation says that the marginal benefit from pollution abatement within jurisdiction $i$ equals the unitary marginal cost (note $U_P < 0$). Since $a_i$ and $a_{-i}$ are arguments of both $U_P$ and $U_{c_i}$, (7) implicitly defines jurisdiction $i$’s reaction function, as in (2) (the $a$’s replace the $z$’s).

To analyze the social optimum, consider the symmetric case, where income takes a common value $y$ for all jurisdictions. Optimal abatement levels are then uniform, and the optimal value maximizes $U(y - a, \tau - \gamma na)$. The first-order condition is

$$-\frac{n\gamma U_P}{U_{c_i}} = 1. \quad (8)$$

The factor of $n$, which is not present in (7), captures the benefits arising in all jurisdictions when any one jurisdiction’s abatement level is increased. Since this spillover effect is not considered in individual decisions, the Nash equilibrium abatement level is smaller than the socially optimal level.

### 3.2. Reaction functions

To analyze reaction functions for this model, suppose that $U$ takes the Cobb-Douglas form $c_i^\rho P^\lambda$, where $\rho > 0$ and $\lambda < 0$. Then, after substituting as above, differentiation with respect to $a_i$ yields the relevant form of (7), which can be solved for $a_i$ to give

$$a_i = \Gamma_i - \frac{\rho}{\rho + \lambda} \sum_{i \neq j} a_j, \quad (9)$$
where $\Gamma_i$ is a constant that depends on $y_i$. Given that $\rho + \lambda < 0$ must hold for the jurisdiction’s second-order condition to be satisfied, it follows that the reaction function in (9) is an upward-sloping function of the $a_j$’s. Since it can be verified that $V_{aiaj} > 0$ holds in the Cobb-Douglas case as long as $\rho \gamma > 0$, this positive slope is consistent with the general analysis above. Note that the abatement level in the symmetric Nash equilibrium is found by replacing the $a$’s in (9) with a common value and solving ($\Gamma$ loses its $i$ subscript under symmetry).

3.3. Other spillover models

Other spillover models involve more-general public expenditure spillovers across jurisdictions. In such models, residents of one jurisdiction effectively consume the public goods provided by other jurisdictions along with those provided by their own governments. The best-known study of this type is Case, Rosen and Hines (1993), who focus on interstate expenditure spillovers. A number of other studies consider spillovers across local jurisdictions. Examples of expenditure spillovers might include the benefits from using highways in an adjacent state, or the benefits from visiting a municipal museum in a nearby city.

Models of “yardstick competition,” exemplified by the work of Besley and Case (1995), involve information spillovers across jurisdictions. In their model, voters look at public services and taxes in other jurisdictions to help judge whether their government is wasting resources (through inefficiency or rent-seeking) and deserves to be voted out of office. Knowing that voters make such comparisons, government officials take other jurisdictions’ choices into account in making their own decisions, leading to strategic interaction.


4.1. Model

To illustrate the resource-flow model, consider a model of welfare competition between the states, following Wildasin (1991) and Brueckner (2000). For simplicity, let the economy contain just two jurisdictions (states), denoted 1 and 2. Each state contains $M$ nonpoor, immobile consumers, referred to as the “rich.” The economy contains $2N$ poor consumers, who work at low-paying jobs as well as receiving welfare benefits from the state where they reside. The poor are assumed to be mobile across states, with $N_1$ and $N_2 = 2N - N_1$ giving the poor
populations in states 1 and 2 respectively. The poor populations correspond to the “resource” that appears in the resource-flow model, although in the present case, this resource turns out to be a “bad” rather than a good.

State output $f(N)$ depends on the amount $N$ of unskilled labor along with other fixed factors. The unskilled wage, earned by the state’s poor residents, is then given by $w(N) \equiv f'(N)$, where $f'$ is the marginal product. Since $f$ is concave, the wage falls as the unskilled labor pool grows, with $w'(N) = f''(N) < 0$. Wages in states 1 and 2 are thus $w_1 = w(N_1)$ and $w_2 = w(N_2)$, and letting $T_1$ and $T_2$ denote the welfare benefits paid to the poor, the total income of a poor resident equals $w(N_1) + T_1$ in state 1 and $w(N_2) + T_2$ in state 2.

Assuming that migration costs are zero, migration equilibrium is achieved when poor income is equalized between the two states. The equilibrium condition is then $w_1+T_1 = w_2+T_2$, or

$$w(N_1) + T_1 = w(2\bar{N} - N_1) + T_2.$$  

(10)

This condition, which shows how the poor-population resource distributes itself conditional on the levels of the decision values $T_1$ and $T_2$, is analogous to equation (4) in the general discussion of the resource-flow model.

By making state 1 more attractive, an increase in $T_1$ causes welfare migrants to flow from state 2 to state 1, raising $N_1$. Differentiating (10),

$$\frac{\partial N_1}{\partial T_1} = -\frac{1}{w'(N_1) + w'(2\bar{N} - N_1)} > 0; \quad \frac{\partial N_1}{\partial T_2} = -\frac{\partial N_1}{\partial T_1} < 0,$$  

(11)

where $w' < 0$ is used. Note that wage adjustment is the force that equilibrates migration flows. The wage falls in state 1 and rises in state 2 as the poor population relocates, eventually restoring the equality of gross incomes.

The level of each state’s welfare benefit is chosen by its rich residents, who care about the well-being of the local poor. The rich in both states have the same quasi-linear utility function, which is written $c_i + U(w_i + T_i)$, $i = 1, 2$, where $c_i$ gives consumption expenditure for the rich in state $i$, and where $U' > 0$ and $U'' < 0$. Note that the rich care about the total
income of a representative poor resident, $w_i + T_i$, which depends on wage income as well as the welfare benefit. Letting $y_i$ denote the rich income in jurisdiction $i$, the budget constraint of a rich resident in state 1 is then given by $c_1 + N_1 T_1/M = y_1$, with an analogous constraint applying in state 2. Note that the welfare cost for each rich resident equals the total outlay, $N_1 T_1$, divided by the number of rich, $M$.

4.2. Benefit choices

The rich in state 1 choose $T_1$ to maximize utility subject to the budget constraint. To analyze this problem, suppose first that the poor were immobile and evenly divided between states, with $N_1 = \bar{N}$. Under this assumption, the first-order condition for the utility-maximization problem is

$$MU'(w_1 + T_1) = \bar{N}. \quad (12)$$

where $w_1 = w(N)$. This equation says that the sum across the rich population of the gains from increasing the income of the poor by a dollar should equal the cost, equal to $\bar{N}$. Eq. (12) characterizes the socially optimal level of welfare benefits, conditional on an equal division of the poor across the states.

Now consider the case where the poor are mobile. In this case, the rich in state 1 choose the welfare benefit taking account of the fact that an increase in $T_1$ raises $N_1$ through migration. The first-order condition for this problem is

$$MU'(w_1 + T_1) = \frac{N_1 + \frac{\partial N_1}{\partial T_1} T_1}{1 + w'(N_1) \frac{\partial N_1}{\partial T_1}}. \quad (13)$$

where $w_1 = w(N_1)$. Recognizing that $N_1$ and hence $w_1$ depend on both $T_1$ and $T_2$ via (10), eq. (13) has the form of the condition $V_{z_i}(z_i, z_{-i}; X_i) = 0$ from the general discussion, with the $T$’s replacing the $z$’s. Thus, (13) implicitly defines state 1’s reaction function, as discussed further below.

Although the left-hand side of (13) again equals the sum of the marginal gains from an increase in the income of the poor, the RHS differs from (11) because of the effects of migration. The numerator of this expression gives the increase in total welfare outlays from an increase
in $T_1$. While the $N_1$ term again captures the effect of the higher benefit per recipient, the additional $(\partial N_1/\partial T_1)T_1$ term (which is positive from (11)) captures the effect of a larger poor population.

To understand the denominator of (13), note that in the immobility case, increasing the welfare benefit by a dollar raises the income of the poor by a dollar. However, with migration, the wage falls as additional poor move into state 1, offsetting the effect of the higher $T_1$, and this effect is captured by the denominator of (13). Using (11) and $w'(N_1) < 0$, it can be seen that the denominator is less than one but greater than zero, which implies that a dollar increase in $T_1$ raises $w_1 + T_1$ but does so by less than a dollar. Migration of the poor thus reduces the “productivity” of welfare benefits in raising poor incomes.

The combination of the above effects reduces the welfare benefit below the socially optimal level. To see this conclusion, note first that since the Nash equilibrium will be symmetric, the wage levels on the LHS (13) will equal $w_1(N)$, the same value as in (12). Next, observe that the RHS of (13), which is evaluated with $N_1 = \bar{N}$, takes on a value greater than $\bar{N}$, given the extra positive term in the numerator and the fact that denominator is less than unity. For (13) to be satisfied, the equilibrium $T_1$ value must then be smaller than the one in (12) (recall $U'' < 0$). Thus, equilibrium welfare benefits are inefficiently low. Because the perceived cost of redistribution is raised by welfare migration, the rich naturally choose to limit their generosity.

4.3. Reaction functions

As noted above, state 1’s reaction function is implicitly defined by the first-order condition (13). Since $N_1$ and thus $w_1$ depend on both $T_1$ and $T_2$ via (10), both sides of (13) depend on both benefit levels. In principle, (13) can then be rewritten to give $T_1$ as a function of $T_2$, yielding state 1’s reaction function.

To derive the reaction for a special case, assume that the production function $f$ is quadratic, so that $f'(N_1)$ and hence $w(N_1)$ is given by the linear function $\alpha - \mu N_1$, where $\alpha, \mu > 0$. Assume that the $U$ portion of the utility function is also quadratic, so that $U'(w_1 + T_1) = \eta - \delta(w_1 + T_1)$, where $\eta, \delta > 0$. Substituting the linear $f'$ in (10) and solving for $N_1$ yields $N_1 = \bar{N} + (T_1 - T_2)/2\mu$. This equation shows that state 1’s poor population is increasing in $T_1$ and decreasing in $T_2$, as noted in (11). Using this equation to substitute for $N_1$ and $\partial N_1/\partial T_1$
in (13), and substituting the above expression for $U'$, (13) can be rewritten as

$$T_1 = \Phi + \frac{2 - \mu \delta M}{4 + \mu \delta M} T_2,$$

(14)

where $\Phi$ is a constant. Equation (14) gives state 1’s reaction function, and state 2’s function is gotten by interchanging $T_1$ and $T_2$. The intersection of these two reaction functions yields the common equilibrium value of $T_1$ and $T_2$, which is inefficiently low. This value is found by setting $T_1 = T_2 = T$ in (14) and solving for $T$.

The reaction function in (14) is linear, and it may slope up or down depending on parameter values. The function is upward sloping if $1/\mu > \delta M$, with state 1 raising $T_1$ in response to an increase in $T_2$. For this condition to hold, the $\mu$ parameter from the production function must be small relative to the $\delta$ parameter from the utility function. Conversely, the function is downward sloping if $1/\mu < \delta M$, with state 1 reducing its welfare benefit in response to an increase in $T_2$. The slope is zero only in the unlikely knife-edge case where $1/\mu = \delta M$.

4.4. Other resource-flow models

The public economics literature contains other models of strategic interaction based on a resource-flow framework. The tax-competition literature, surveyed by Wilson (1999), offers the largest set of examples. In such models, local governments finance public spending with a tax on mobile capital, which plays the role of the resource (now a “good” rather than a “bad”). When a jurisdiction raises its tax rate, the net-of-tax return is driven below the level prevailing elsewhere in the economy, and capital relocates until net returns are once again equalized. This capital flight reduces a jurisdiction’s incentive to raise its tax rate, leading to underprovision of public goods.

When jurisdictions are large, this tax-setting behavior involves strategic interaction, with tax rates elsewhere affecting a given jurisdiction’s decision. As in the welfare competition model, reaction functions under tax competition can slope up or down, with the direction depending on the parameterization of preferences and production.
5. Testing for Strategic Interaction

As seen above, both the spillover and resource-flow models of strategic interaction generate reaction functions, which relate each jurisdiction’s chosen $z$ to its own characteristics and to the choices of other jurisdictions. The goal of empirical work is to test for strategic interaction by estimating such functions.

The previous discussion showed that, when strategic interaction occurs, the reaction function slope is nonzero except in unlikely cases. As a result, a proper test for interaction involves a simple significance test on the slope coefficient. If the null hypothesis of a zero reaction-function slope can be rejected, then the evidence points to the existence of strategic interaction among jurisdictions.

Assuming linearity of the reaction function, and following (2), an appropriate estimating equation can be written

$$z_i = \beta \sum_{j \neq i} \omega_{ij} z_j + X_i \theta + \epsilon_i,$$

(15)

where $\beta$ and $\theta$ are unknown parameters (the latter a vector), $\epsilon_i$ is an error term, and the $\omega_{ij}$ represent nonnegative weights, which are specified a priori. These weights indicate the relevance of other jurisdictions $j$ in the process of interaction, and they can be viewed as part of jurisdiction $i$’s characteristics. The weights typically capture the location of $i$ relative to other jurisdictions, and a scheme that assigns weights based on contiguity is commonly used. Under such a scheme, $\omega_{ij} = 1$ for jurisdictions $j$ that share a border with $i$ and $\omega_{ij} = 0$ for noncontiguous jurisdictions. Thus, for example, in Saavedra’s (2000) study of welfare competition, nonzero weights are given to states adjacent to a given state, while nonbordering states received a weight of zero. Generally, the weights are normalized so that their sum equals unity for each $i$. Thus, in Saavedra’s study, if a state has four states adjacent to it, each receives a weight of $1/4$.

As explained above, $X_i$ in (15) is a vector of characteristics for jurisdiction $i$. In Saavedra’s paper, $X_i$ includes the per capita income of the given state, the African-American proportion of its population, its female unemployment rate, a measure of the political makeup of the state’s legislature, and other variables. The dependent variable $z_i$ in her study is the dollar
welfare payment for a family of size three.

While choice of the weights in (15) is based on prior judgement about the pattern of interaction, the parameter $\beta$, which reflects the strength of interaction among jurisdictions, is estimated from the data. Note that (15) implies that the direction of $i$’s interaction with all other jurisdictions is the same, with its sign determined by the sign of $\beta$. The magnitude of the effect, however, depends on the relevant weight, with $\partial z_i / \partial z_j = \beta \omega_{ij}$.

As is well known from the literature on spatial econometrics (see Anselin (1988)), two main econometric issues must be confronted in estimating (15). These are endogeneity of the $z_j$’s, and possible spatial error dependence. These issues are considered in turn in the following discussion.

\subsection*{5.1. Endogeneity of the $z_j$’s}

Because of strategic interaction, the $z$ values in different jurisdictions are jointly determined. As a result, the linear combination of the $z_j$’s appearing on the RHS of (15) is endogenous and correlated with the error term $\epsilon_i$. To see this point formally, the first step is to rewrite (15) in matrix form, which yields

$$
z = \beta W z + X \theta + \epsilon,
$$

where $z$ is the vector of the $z_i$’s, $X$ is the characteristics matrix, and $W$ is the weight matrix, with representative element $\omega_{ij}$ (note that $\omega_{ii} = 0$ for all $i$). Then, (16) can be used to solve for the equilibrium values of the $z_i$’s, which yields

$$
z = (I - \beta W)^{-1} X \theta + (I - \beta W)^{-1} \epsilon.
$$

The key implication of (17) is that the random component of $z_k$ is equal to the inner product of the $k$th row of the matrix $(I - \beta W)^{-1}$ and the error vector $\epsilon$. Each element of $z$ thus depends on all the $\epsilon$’s. As a result, each of the $z_j$’s on the RHS of (15) depends on $\epsilon_i$, the equation’s error term. The resulting correlation means that OLS estimates of the parameters of (15) are inconsistent, requiring use of an alternate estimation method.
To understand the correlation between the $z_j$'s and $\epsilon_i$ on a more intuitive level, consider the case of just two jurisdictions, 1 and 2. In this case, only one $z_j$ appears on the RHS of the reaction function in (15), and the matrix equation (16) consists of just two reaction functions, one for each jurisdiction. The solution in (17) then corresponds to the intersection of these reaction functions in two-dimensional space. It is clear that the $z_1$ value at this intersection point depends on the magnitudes of the intercepts of both reaction functions, as does the value of $z_2$. But observe that the reaction-function intercept itself depends on the magnitude of the jurisdiction’s error term. When $\epsilon_1$ increases, jurisdiction 1’s reaction function shifts up in a parallel fashion, and an increase in $\epsilon_2$ has an analogous effect. The upshot is that the value of $z_1$ at the intersection point depends on both $\epsilon_1$ and $\epsilon_2$, as does the value of $z_2$. Thus, $z_2$ on the RHS of jurisdiction 1’s reaction function depends on $\epsilon_1$, the function’s error term, and $z_1$ on the RHS of jurisdiction 2’s function depends on $\epsilon_2$. Exactly the same conclusion holds with a larger number of jurisdictions, establishing that the $z_j$’s on the RHS of (15) are correlated with the error term.

In the literature, two methods are used to address this econometric problem. Under the first method, the reduced-form equation given by (17) is estimated using maximum likelihood (ML) methods. Note that since the key parameter $\beta$ enters nonlinearly in this equation, a nonlinear optimization routine must be used to estimate it.

The second method is an instrumental variables (IV) approach. Under this approach, the offending $z_j$’s on the RHS of (15) are replaced by fitted values from a first-stage regression. Since these fitted values are asymptotically uncorrelated with the error term, OLS then yields consistent estimates of the reaction-function parameters. A typical IV procedure is to regress $Wz$ on $X$ and $WX$, and to use the fitted values $\hat{Wz}$ as instruments for $Wz$. Note that this procedure involves regressing the weighted linear combination of the $z_j$’s from the RHS of (15) on $X_i$ and on the same linear combination of the $X_j$’s.

5.2. **Spatial error dependence**

The presence of spatial dependence in the errors also complicates the estimation of (15).
When spatial error dependence is present, the error vector $\epsilon$ satisfies the relationship

$$\epsilon = \phi M \epsilon + v,$$  \hspace{1cm} (18)

where $M$ is a weight matrix, which is often assumed to be the same as $W$ in (16), $v$ is a well-behaved error vector, and $\phi$ is an unknown parameter. Solving (18) yields

$$\epsilon = (I - \phi M)^{-1} v,$$  \hspace{1cm} (19)

which shows that each element of $\epsilon$ is a linear combination of the elements of $v$, implying that $\epsilon_i$ is correlated with $\epsilon_j$ for $i \neq j$.

Such spatial error dependence arises when $\epsilon$ includes omitted jurisdiction characteristics that are themselves spatially dependent. For example, suppose that $z$ measures the park acreage in a community, and suppose that such acreage is inversely related to a community’s innate topographical amenities, which may affect its willingness to invest in parks. For example, if a community lies on the ocean or has attractive mountains nearby, then it may be less willing to invest public funds in park acreage than a community without such natural amenities. Since topographical amenities may be unobservable in the data, the amenity level may thus be part of the error term $\epsilon$ in (15). Note that since a high amenity level implies a low $z$, amenities and $\epsilon$ are inversely related.

The source of trouble in this setup is that topographical amenities, and hence the error terms in $\epsilon$, are likely to be spatially correlated. In other words, a high amenity level in one community may imply a high level in nearby communities, with low amenities also associated with low amenities nearby. For example, if one community has a nearby beach or mountains, then its neighbors will probably enjoy the same amenities. If one community lacks amenities, its neighbors are likely to lack them as well. Such a pattern implies spatial error dependence.

When this spatial dependence is ignored, estimation of (15) can provide false evidence of strategic interaction. To understand this outcome, suppose that $\beta = 0$, so that strategic interaction in the choice of park acreage is actually absent. Then, note from above that $\epsilon$ (and
hence $z$) will be low in communities with high natural amenities, while $\epsilon$ and $z$ will be high in communities with poor amenities. But since communities of each type will tend to be near one another because of spatial dependence in the errors, estimation of (15) will indicate a positive association between the $z$ levels in nearby communities, yielding a positive estimate of $\beta$. This result, however, reflects spatial error dependence and not strategic interaction.

To deal with this problem, one approach is to use maximum likelihood to estimate (15), taking account of the error structure in (18). This approach, however, is computationally challenging. In addition, the similar roles played by the parameters $\beta$ and $\phi$ in the model may lead to difficulties in identifying their individual magnitudes (see Anselin (1988)).

An easier remedy is to rely on the IV estimation method discussed above. Kelejian and Prucha (1998) show that this method generates a consistent estimate of $\beta$ even in the presence of spatial error dependence. To see how the IV approach achieves consistency, eliminating false evidence of strategic interaction, the following discussion is useful. First, observe that, if the true $\beta$ equals zero, indicating no strategic interaction, then the $z$’s are not jointly determined, and this source of correlation between the $z_j$’s and the error term in (15) is eliminated. However, with spatial error dependence, a new source of correlation emerges. The problem is that, while a particular $z_k$ on the RHS of (15) is affected only by its own error term $\epsilon_k$, that error term is correlated with $\epsilon_i$ as result of spatial error dependence. Thus, $z_k$ is correlated with $\epsilon_i$, so that estimation of (15) yields inconsistent parameter estimates. By using a fitted value of $z_k$, the IV method purges this correlation and generates consistent estimates.

A third approach is to estimate (15) by ML under the assumption that spatial error dependence is absent, relying on hypothesis tests to verify this absence. Because a test based on the ML results themselves is invalid if spatial dependence is actually present, the robust tests of Anselin, Bera, Florax and Yoon (1996) can be employed instead. These tests are based on OLS estimates of (15), and they are not contaminated by uncorrected spatial error dependence.

5.3. Empirical studies

A growing empirical literature is directed toward testing for strategic interaction among governments. Most studies estimate an equation like (15), usually taking account of the econometric issues discussed above. Among studies based on the spillover model, Murdoch, Sandler


6. Conclusion

This chapter paper has provided an overview of conceptual issues in the literature on strategic interaction among governments, focusing on both the theoretical and empirical sides of the literature. The discussion has argued that most theoretical models can be classified as either spillover or resource-flow models, and the structure of a representative model of each type has been explained in detail. In addition, the chapter has discussed the main econometric issues that arise in testing for strategic interaction and outlined the methods for dealing with them.

While the path from a theoretical model to an empirical reaction function may be clear, as seen in the above discussion, it is important to recognize that the reverse path may involve ambiguity. To understand this point, suppose that empirical results provide evidence of strategic interaction among jurisdictions in the choice of a particular policy variable. The question then is: what underlying jurisdictional behavior is likely to have generated the observed interaction? This issue can be seen most clearly in the choice of tax rates. Strategic interaction in choosing tax rates could arise through the tax-competition model, with jurisdictions concerned about the loss of tax base to neighboring jurisdictions. But interaction could also arise through the yardstick-competition model, with government officials looking at neighboring jurisdictions to
avoid setting tax rates at levels that will displease the voters.

Several papers in the literature address this ambiguity by providing auxiliary evidence to uncover the source of observed strategic interaction. For example, to substantiate the comparative behavior underlying the yardstick-competition model, Besley and Case (1995) estimate an auxiliary equation that relates voter approval of an incumbent to taxes in neighboring jurisdictions (expecting a positive coefficient). Similarly, to support their view that tax-competition behavior underlies interaction in the choice of tax rates, Brett and Pinkse (2000) estimate an equation relating a jurisdiction’s tax base to its tax rate (expecting a negative coefficient). By illuminating the source of the interaction, such auxiliary evidence lends credibility to claims, based on reaction-function estimates, that governments engage in strategic behavior.
References


